

Superconductivity - Overview

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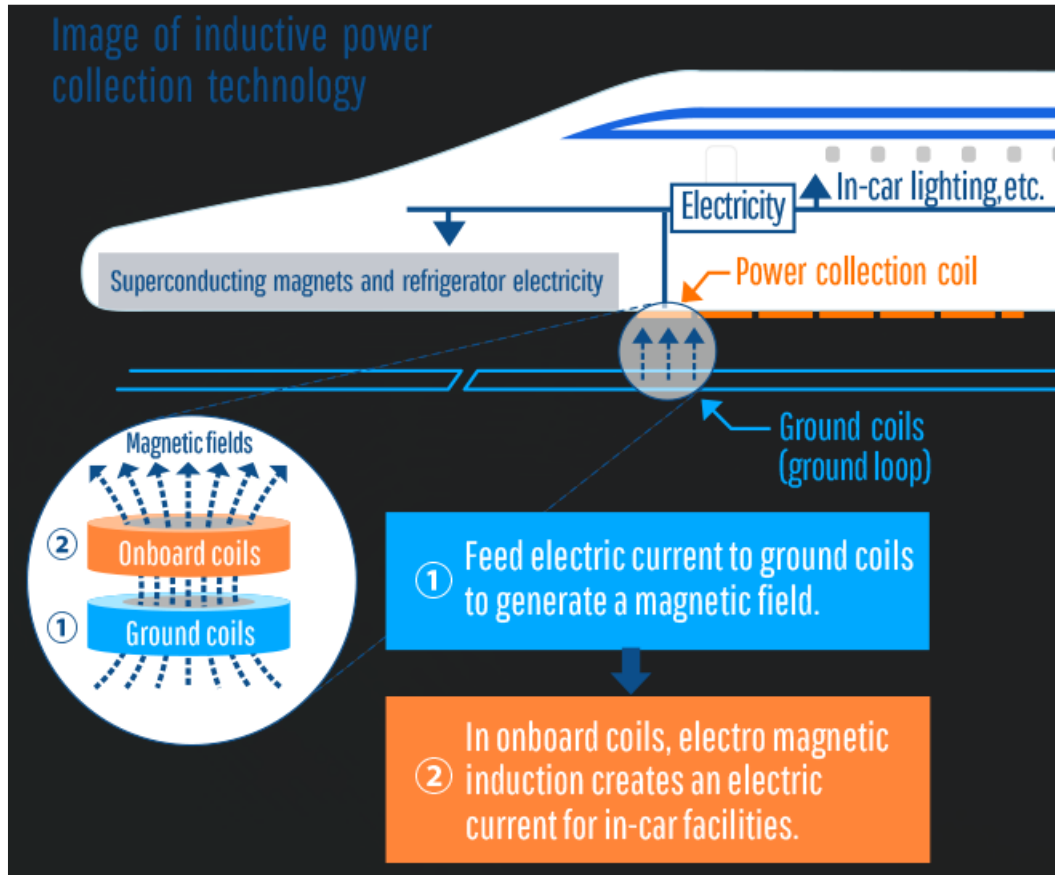
Importance of superconductivity: Qubits and modern quantum computers are made of superconductors

IBM Q



TM

Superconducting-magnet levitation train



The [L0 Series](#), a prototype vehicle based on SCMaglev technology, holds the record for fastest crewed rail vehicle with a record speed of 603 km/h (375 mph).

Time Urbana-Chicago: only ~25 min

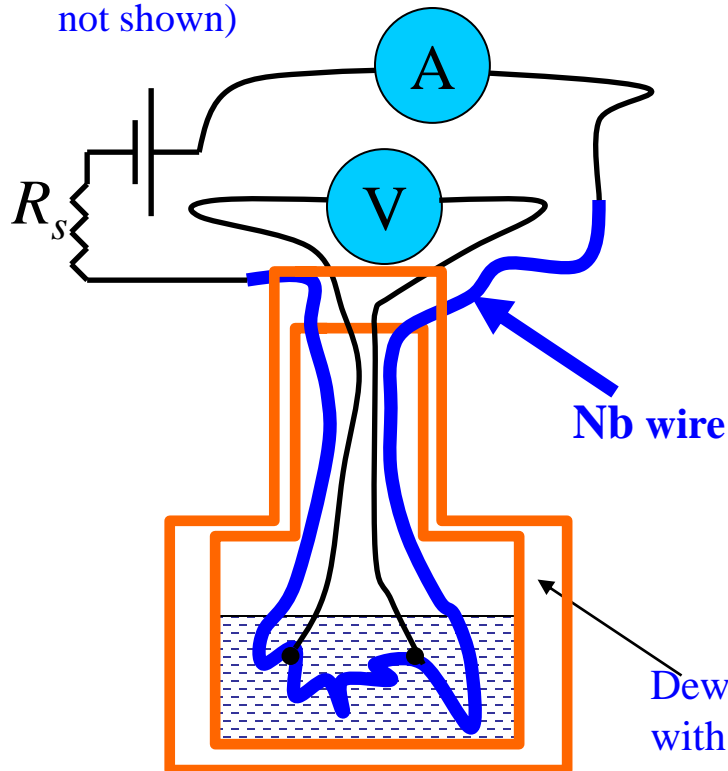
The SCMaglev system uses an [electrodynamic suspension](#) (EDS) system. The train's [bogies](#) have [superconducting](#) magnets installed, and the guideways contain two sets of metal coils.

How to measure superconducting transitions

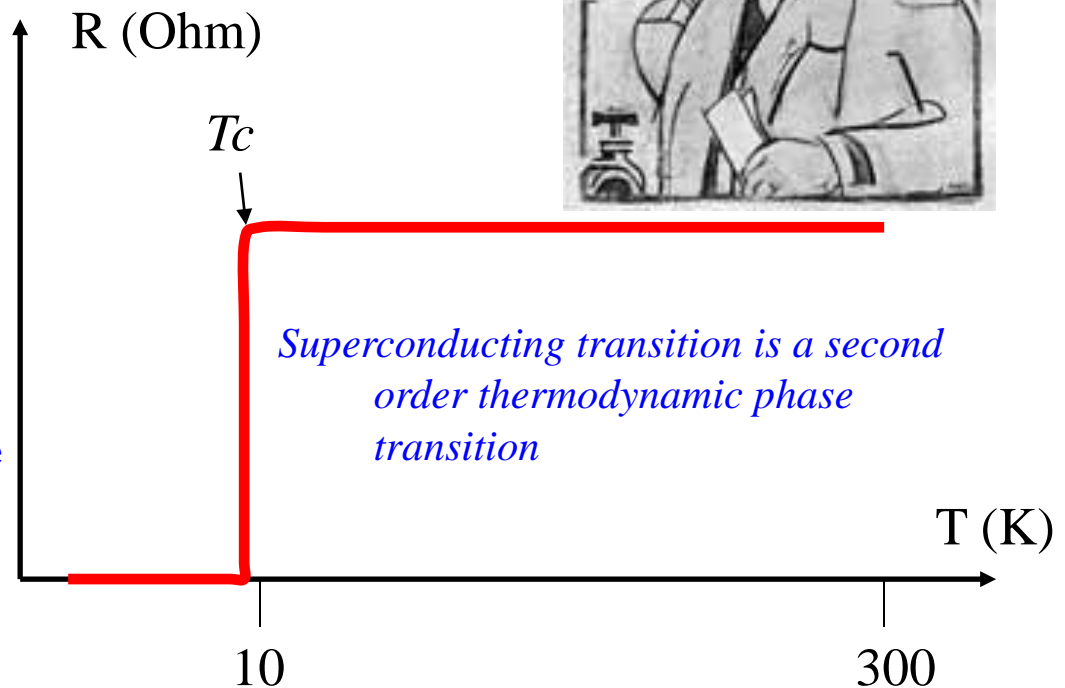
Electrical resistance of some metals drops to zero below a certain temperature which is called "critical temperature" (H. K. O. 1911)

How to observe superconductivity

1. Take Nb (niobium) wire
2. Connect to a voltmeter and a current source
3. Immerse into helium Dewar (T=4.2 K boiling point)
4. Measure electrical resistance (R) versus the temperature (T) (thermometer is not shown)

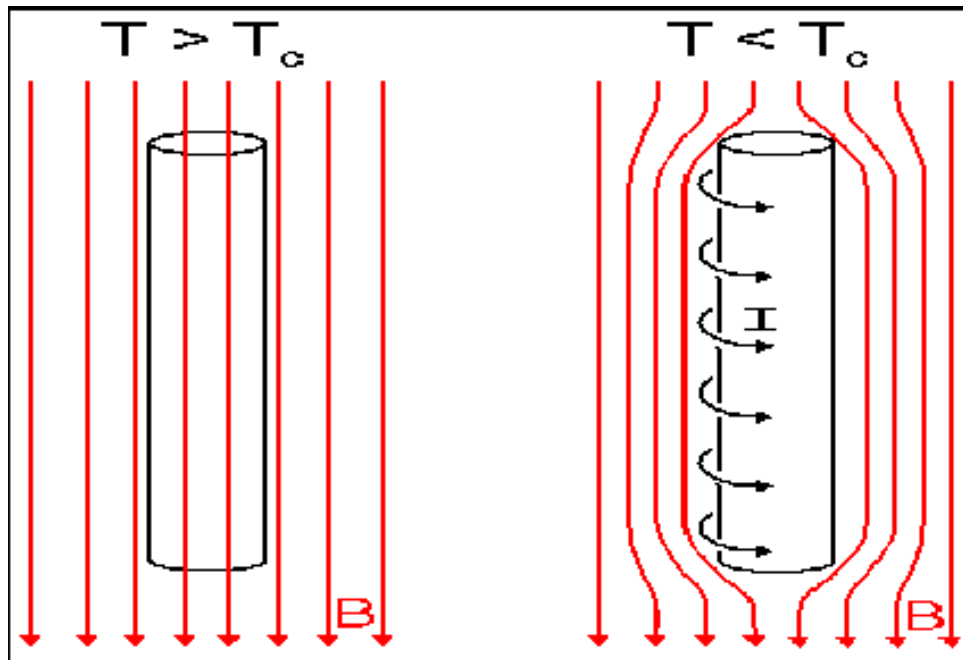


Heike Kamerling Onnes





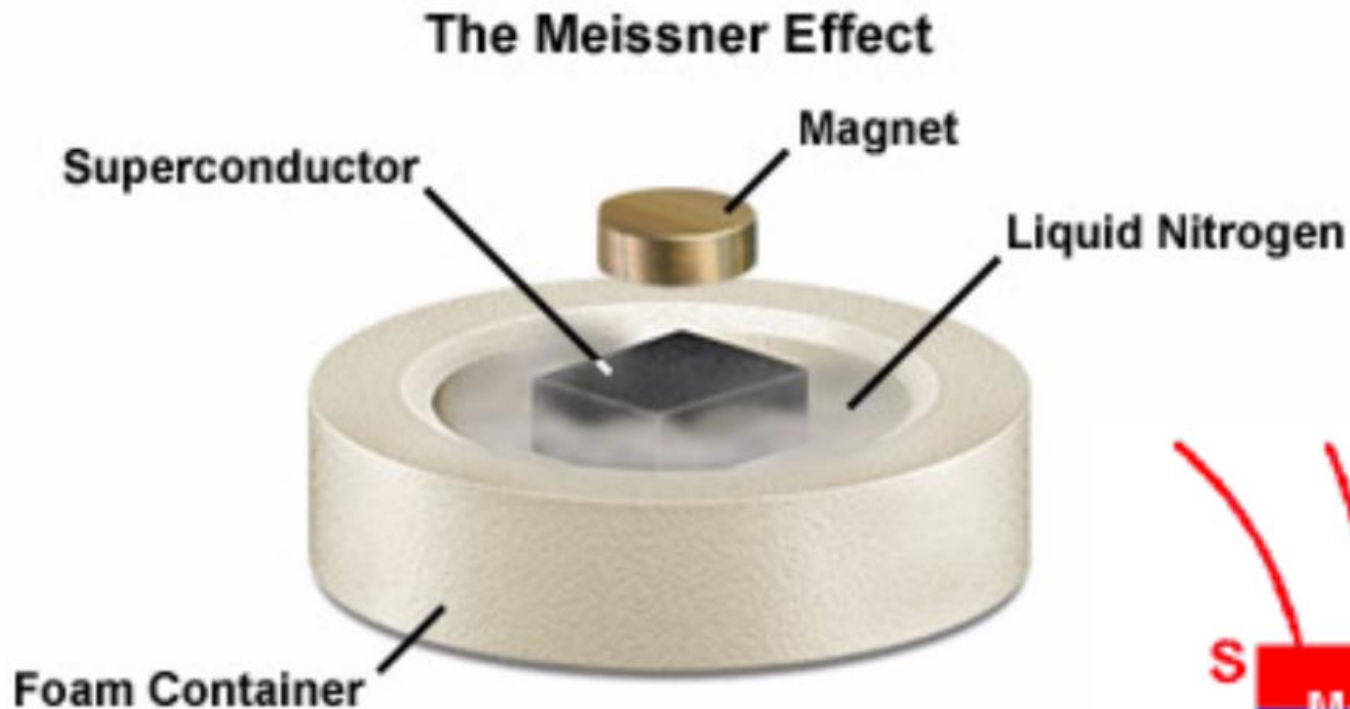
Meissner effect – the key signature of superconductivity



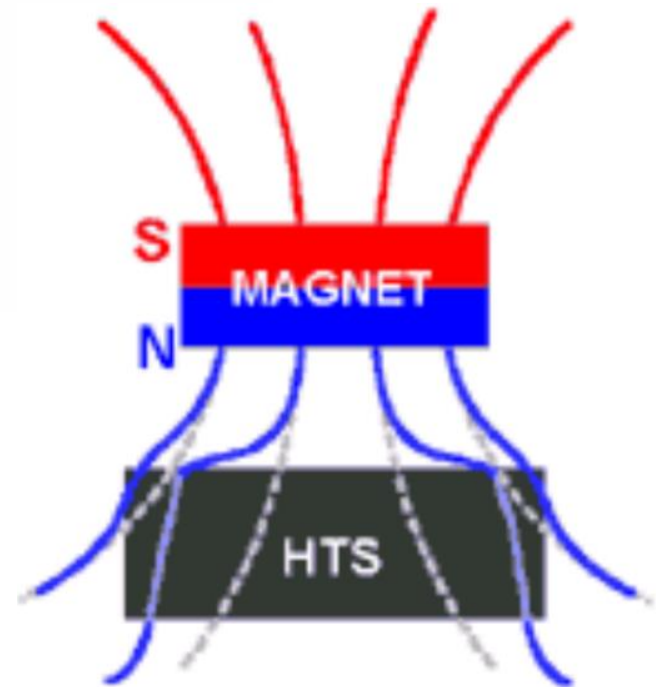
Theory of superconductivity:
 “BCS” – due to Bardeen, Cooper and Schrieffer

Formula	T_c (K)	H_c (T)	Type	BCS
Elements				
Al	1.20	0.01	I	yes
Cd	0.52	0.0028	I	yes
Diamond:B	11.4	4	II	yes
Ga	1.083	0.0058	I	yes
Hf	0.165		I	yes
α -Hg	4.15	0.04	I	yes
β -Hg	3.95	0.04	I	yes
In	3.4	0.03	I	yes
Ir	0.14	0.0016 ^[7]	I	yes
α -La	4.9		I	yes
β -La	6.3		I	yes
Mo	0.92	0.0096	I	yes
Nb	9.26	0.82	II	yes
Os	0.65	0.007	I	yes

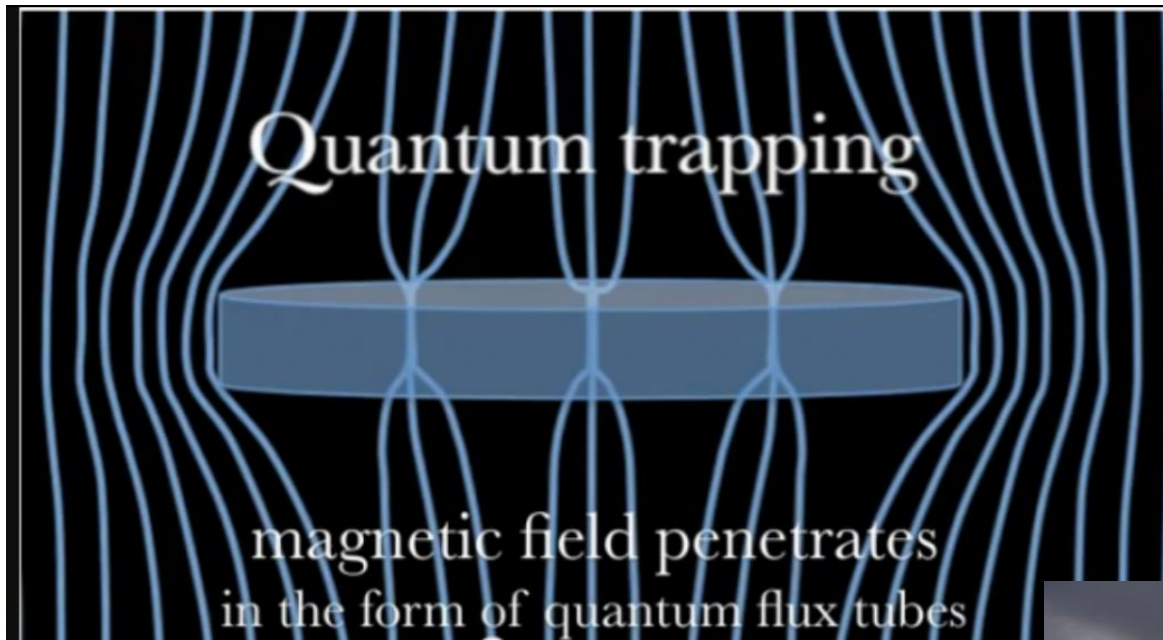
Interesting phenomenon: Magnetic levitation



Levitation is the process by which an object is held aloft, without mechanical support, in a stable position.



Magnetic field effect: Superconducting vortices

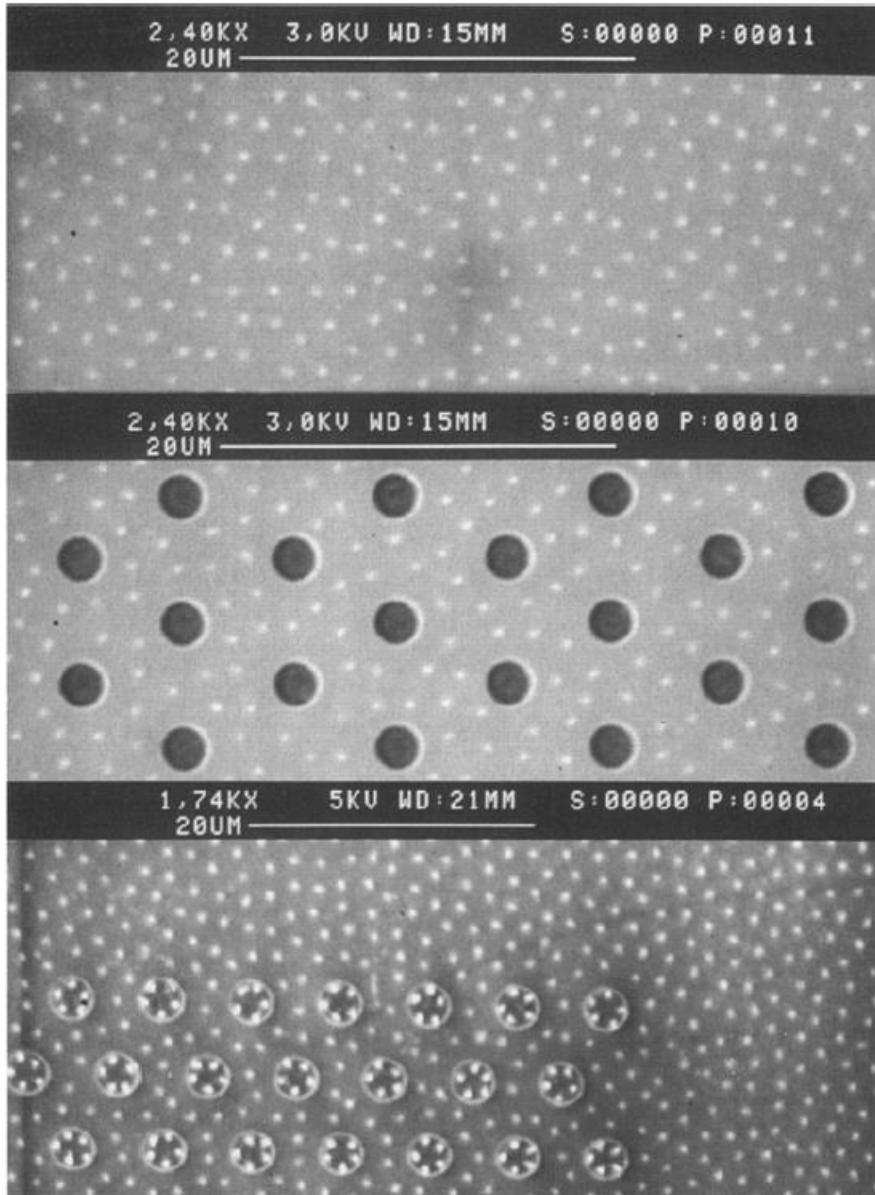


In superconductivity, a fluxon (also called an **Abrikosov vortex** or **quantum vortex**) is a vortex of supercurrent in a type-II superconductors

<https://blog.tmcnet.com/blog/tom-keating/technology-and-science/quantum-levitation-back-to-the-future-hoverboard.asp>



Vortices in superconducting films with “through” and “blind” holes (“antidots”)



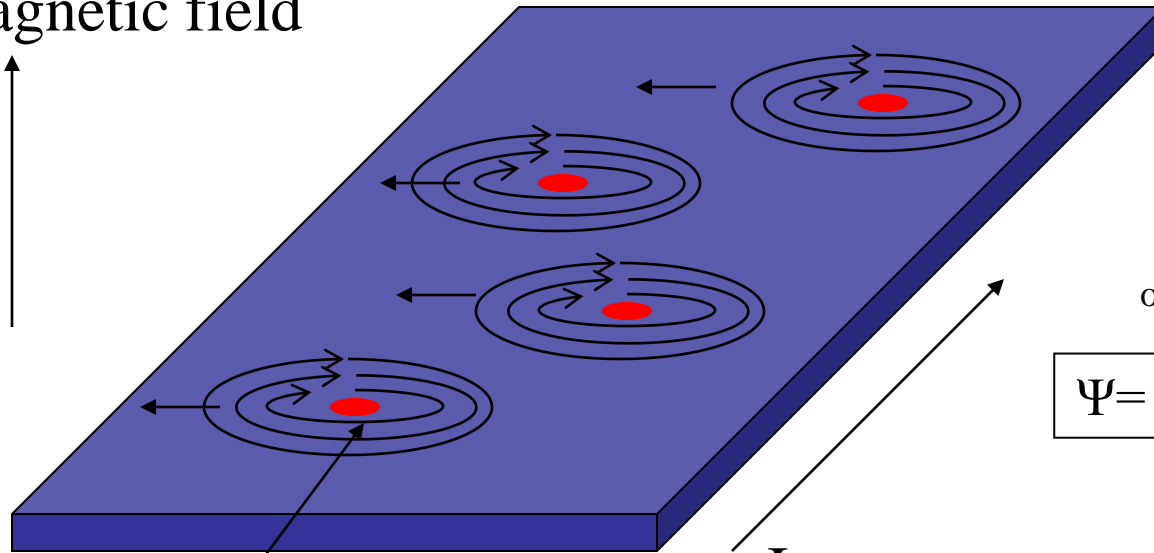
A. Bezryadin and B. Pannetier
“Role of Edge Superconducting States in
Trapping of Multi-Quanta Vortices by
Microholes. Application of the Bitter
Decoration Technique”,
J. Of Low Temp. Phys., V.102, p.73 (1996).

Vortices are quantized tubes carrying magnetic field into superconductor

Magnetic field creates vortices--

Vortices cause dissipation (i.e. a non-zero electrical resistance), if they move

B -magnetic field



Wave function
of all superconducting electrons:

$$\Psi = |\Psi| \exp(i\phi) = |\Psi| \exp(i\theta)$$

amplitude

phase

Vortex core (red): normal, not superconducting; diameter $\xi \sim 10$ nm

The current is extended to a scale λ , which is larger than ξ in type II superconductors (such as thin films of any material)

Reminder: single electron in empty space

Wave function: $\Psi = |\Psi| \exp(ikx) = |\Psi| [\cos(kx) + i \sin(kx)]$

Wave number: $k = 2\pi/\lambda$

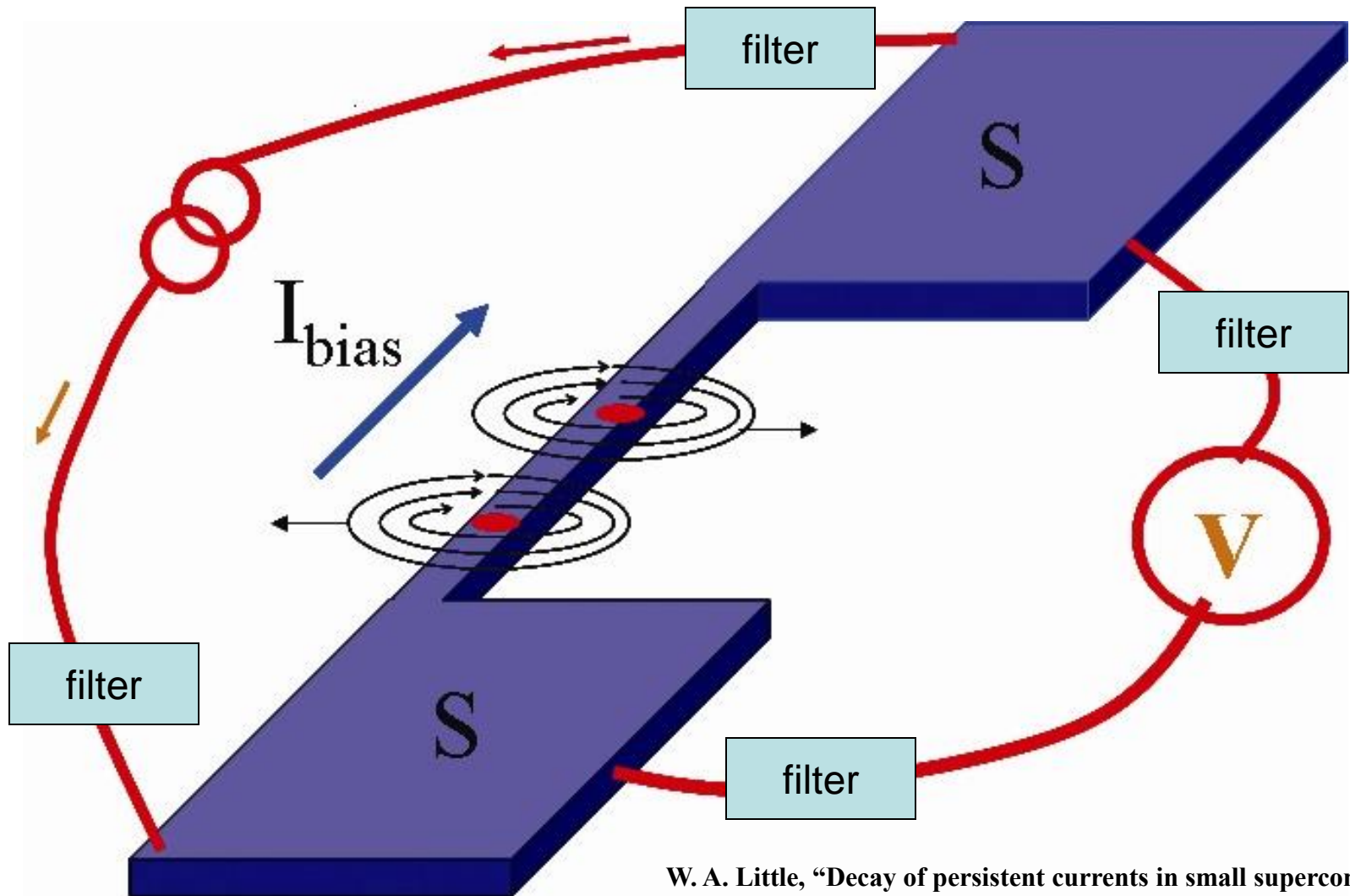
$$i * i = -1$$

General form: $\Psi = |\Psi| \exp(-i\phi)$

In this example of a plane wave, the phase is: $\phi = kx$

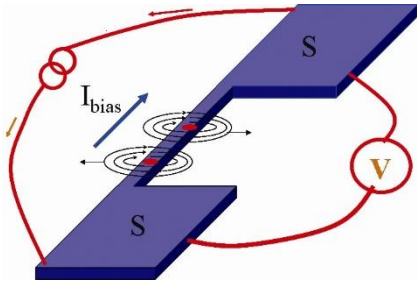
DC transport measurement schematic to detect passing vortices

Bleu: superconducting film and wire
Red: Phase slip events or crossing vortices



W. A. Little, "Decay of persistent currents in small superconductors",
Physical Review, V.156, pp.396-403 (1967).

How to use voltage to determine the rate of phase slips?



Key principle: every time a vortex crosses the wire the phase difference changes by 2π .

Phase evolution equation: $d\phi/dt = 2eV/\hbar$

Simplified derivation:

1. Time-dependent Schrödinger equation with fixed energy:

$$\underline{\hspace{10em}} i\hbar(d\Psi/dt) = E \Psi$$

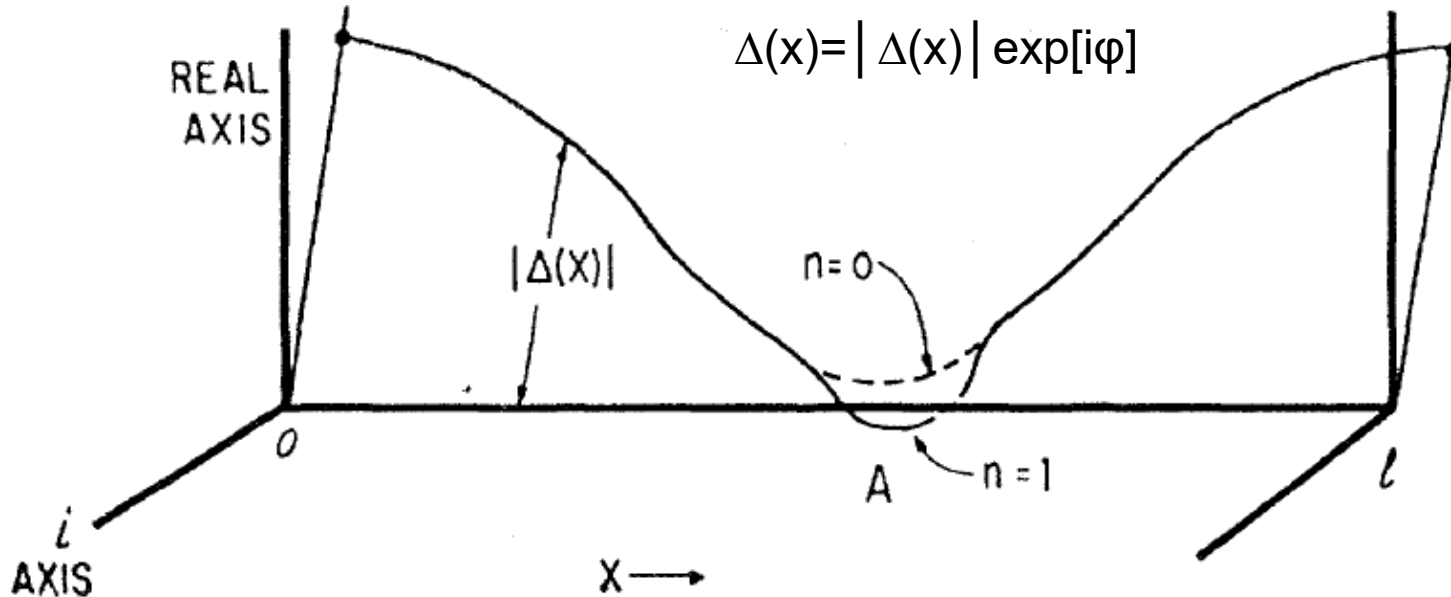
2. The solution is: $\Psi = \exp(-iEt/\hbar)$ (here E is the energy)

3. The phase of the wavefunction is $\phi = Et/\hbar$

4. The energy is defined by the electric potential (voltage), V as follows: $E = 2eV$. Note that the effective charge of superconducting electrons is $2e$, where “e” is the charge of one electron. Such superconducting electron pairs are called Cooper pairs.

Thus, the resulting equation is: $d\phi/dt = 2eV/\hbar$

Thin superconducting wire have some nonzero electrical resistance due to **Little's Phase Slips**



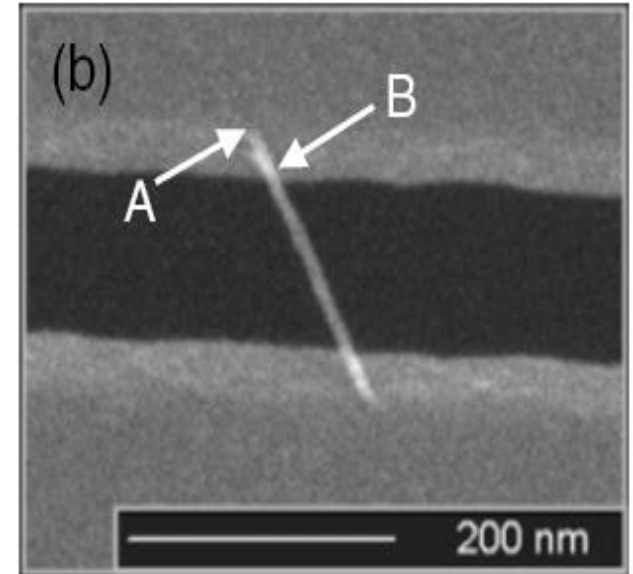
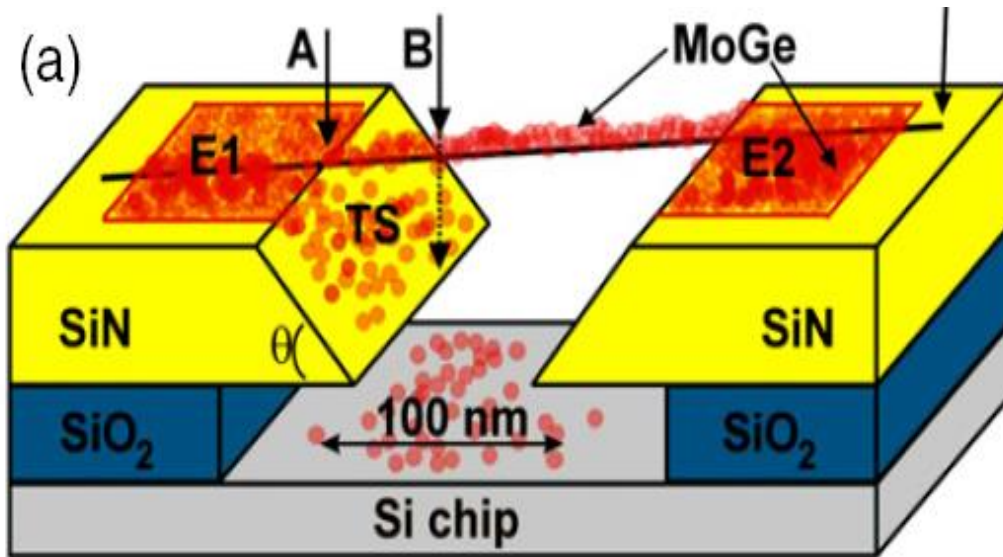
W. A. Little, "Decay of persistent currents in small superconductors", *Physical Review*, V.156, pp.396-403 (1967).

Two types of phase slips (PS) can occur:

1. The usual, thermally activated PS (TAPS)
2. Quantum phase slip (QPS)

Fabrication of nanowires

Method of Molecular Templating



Si/ SiO₂/SiN substrate with undercut

~ 0.5 mm Si wafer

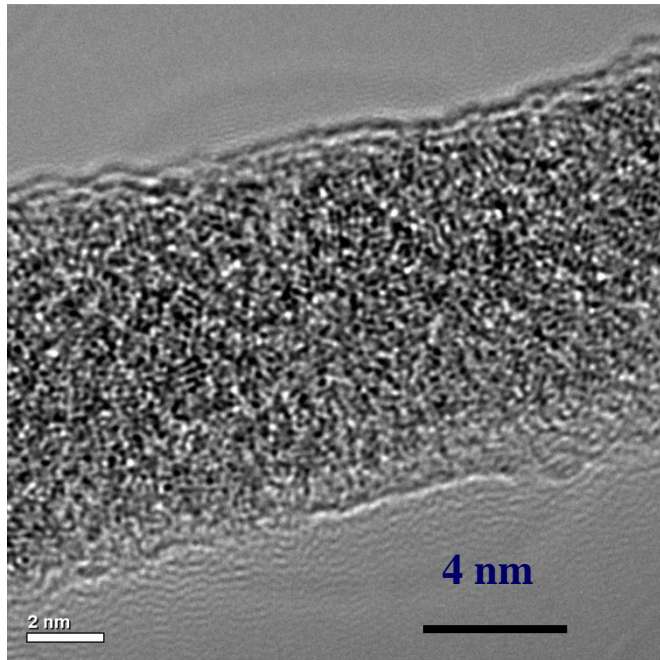
500 nm SiO₂

60 nm SiN

Width of the trenches ~ 50 - 500 nm

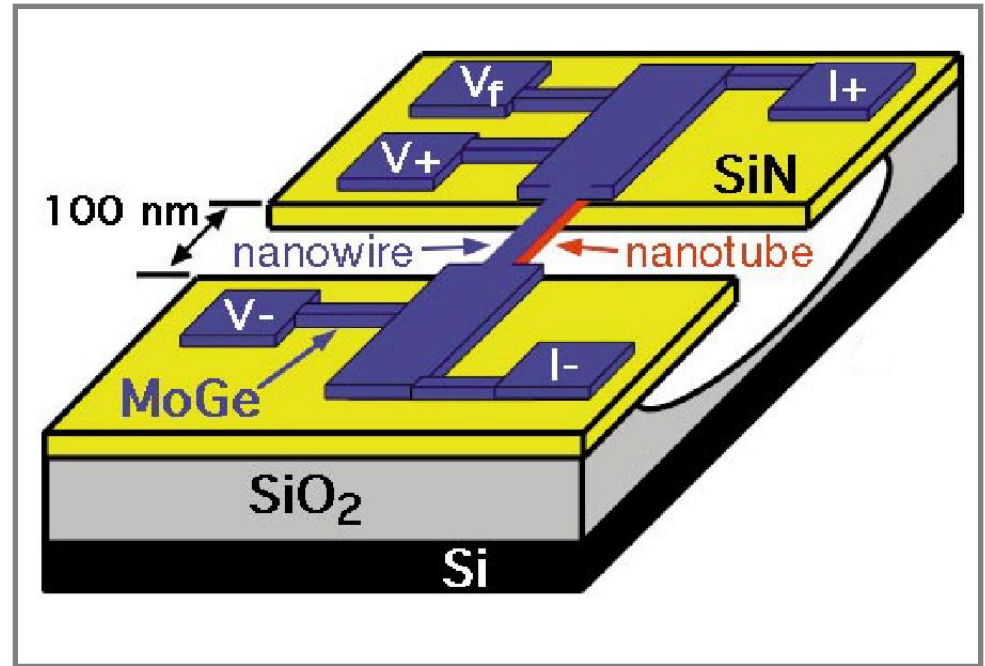
**HF wet etch for ~10 seconds
to form undercut**

Sample Fabrication



TEM image of a wire shows amorphous wire morphology.

Nominal MoGe thickness = 3 nm



**Schematic picture of the pattern
Nanowire + Film Electrodes used in
transport measurements**



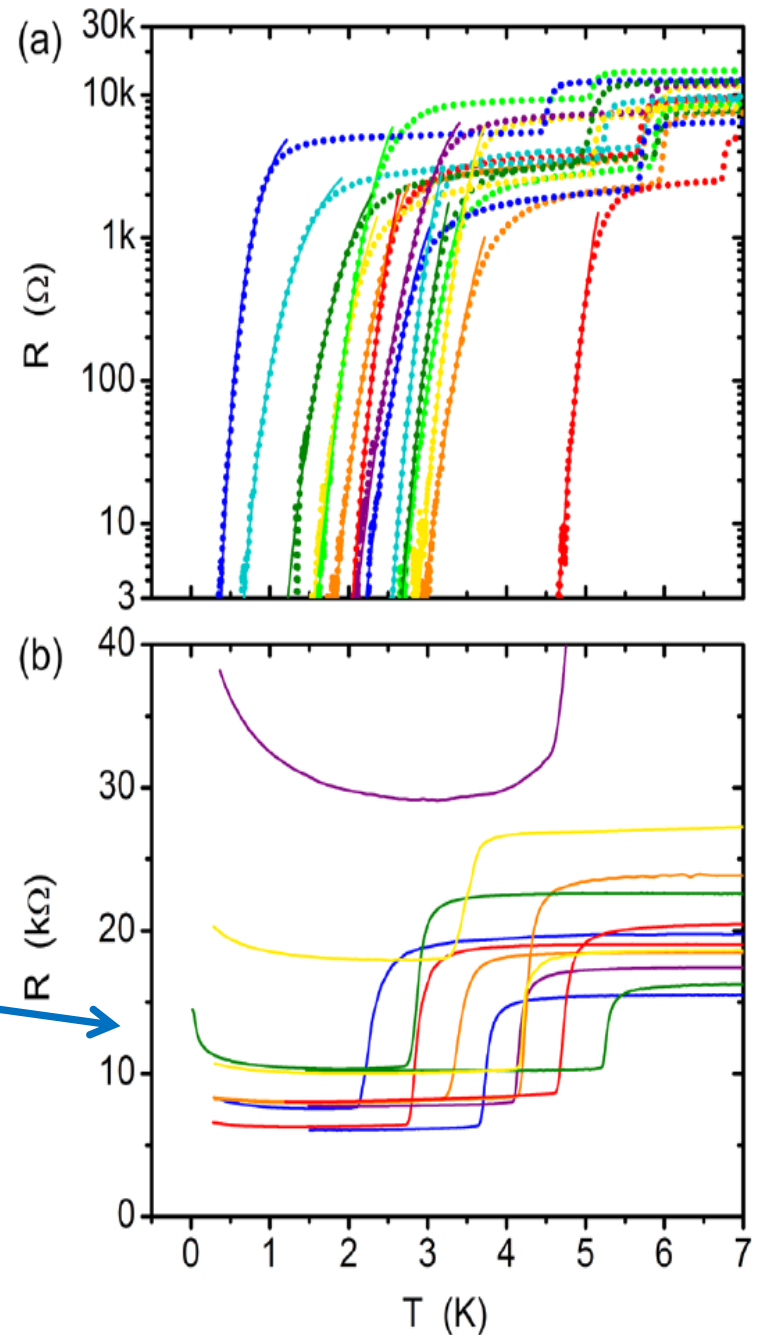
Dichotomy in nanowires: Evidence for superconductor- insulator transition (SIT)

$$R=V/I \quad I \sim 3 \text{ nA}$$

The difference between samples is the amount of the deposited Mo79Ge21.

Thin wires become insulating if their normal resistance is larger than resistance quantum $h/4/e^2 = 6.5 \text{ k}\Omega$

The insulating behavior is due to proliferation of quantum phase slips

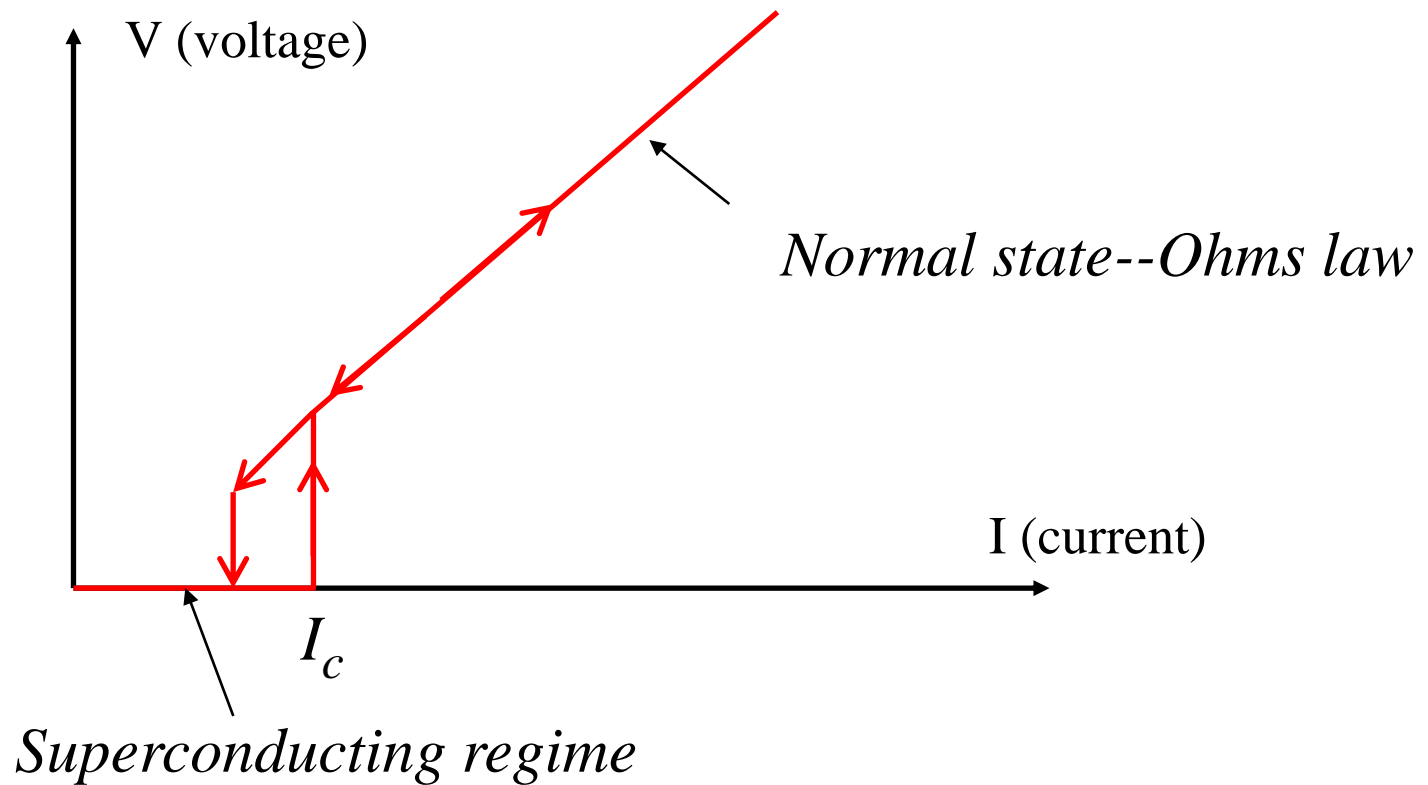


Bollinger, Dinsmore, Rogachev, Bezryadin,
Phys. Rev. Lett. **101**, 227003 (2008)



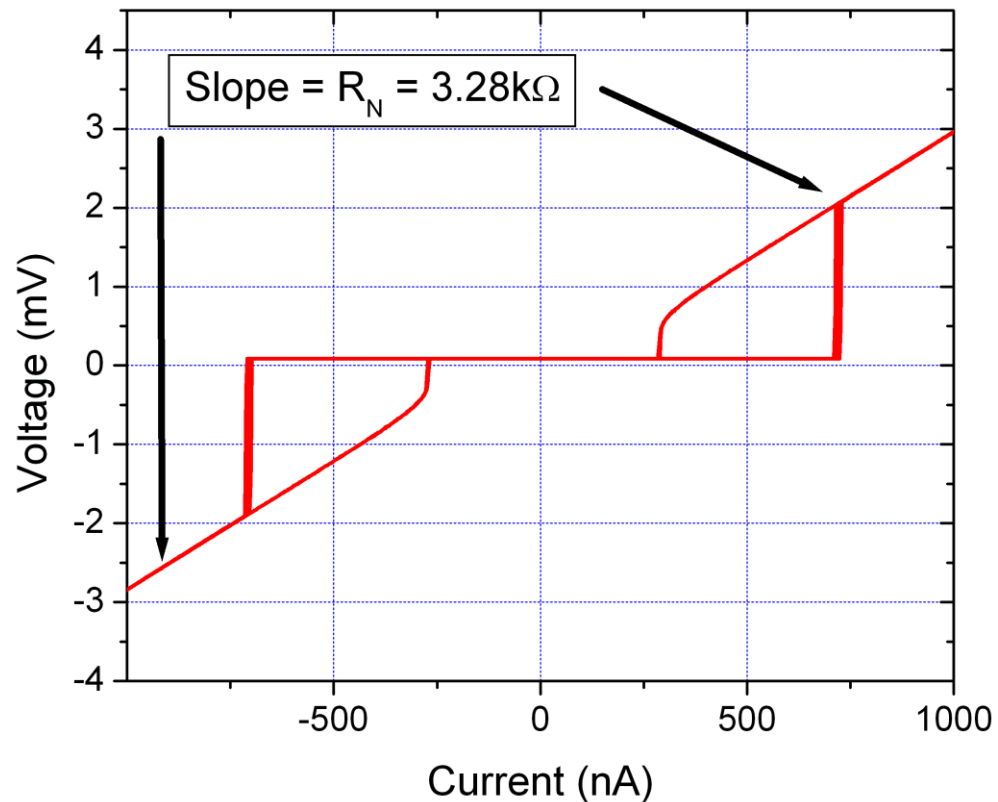
Expected voltage-current curve

Electrical resistance is zero only if current is not too strong



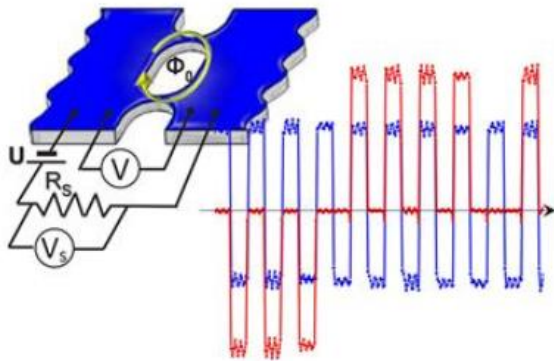
Experimental voltage-current curve.

Fluctuations of the switching current are due to Little's phase slips



Superconducting nanowire memory

scitation.org/journal/apl

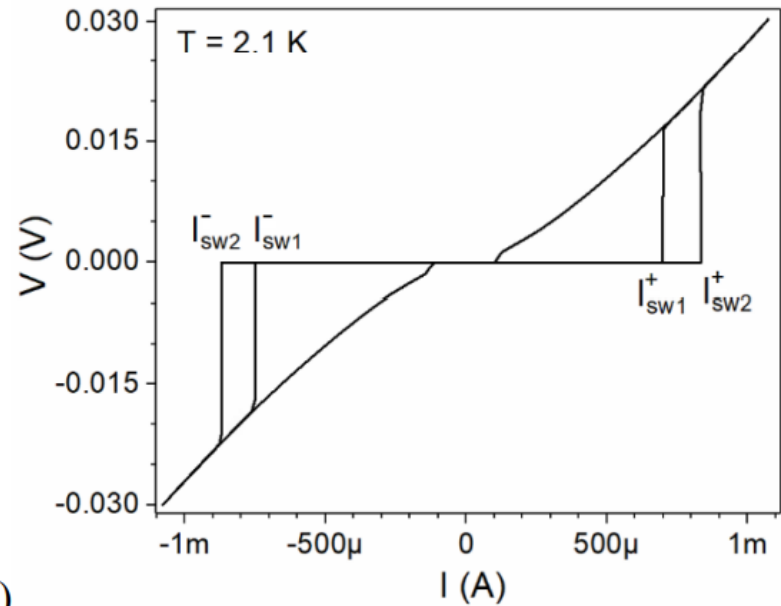
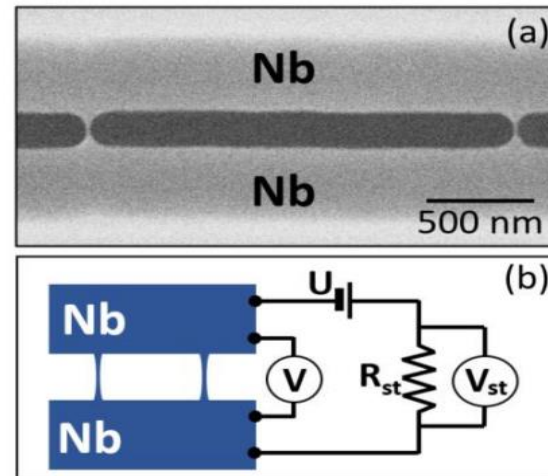


Volume 118, Issue 11, 15 Mar. 2021

Supercurrent-controlled kinetic inductance superconducting memory element

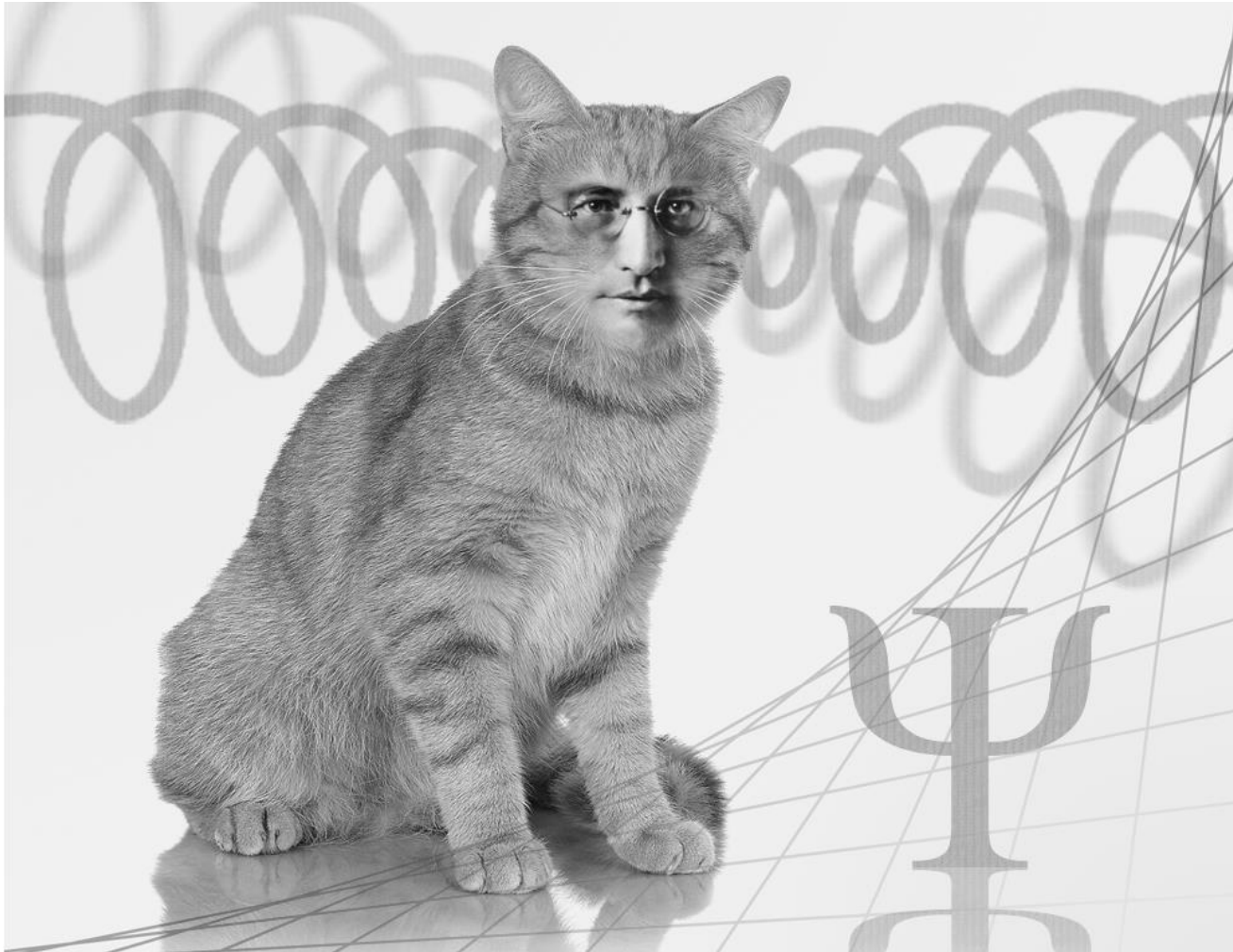
Appl. Phys. Lett. 118, 112603 (2021); doi: 10.1063/5.0040563

Eduard Ilin, Xiangyu Song, Irina Burkova, Andrew Silge, Ziang Guo, Konstantin Ilin, and Alexey Bezryadin

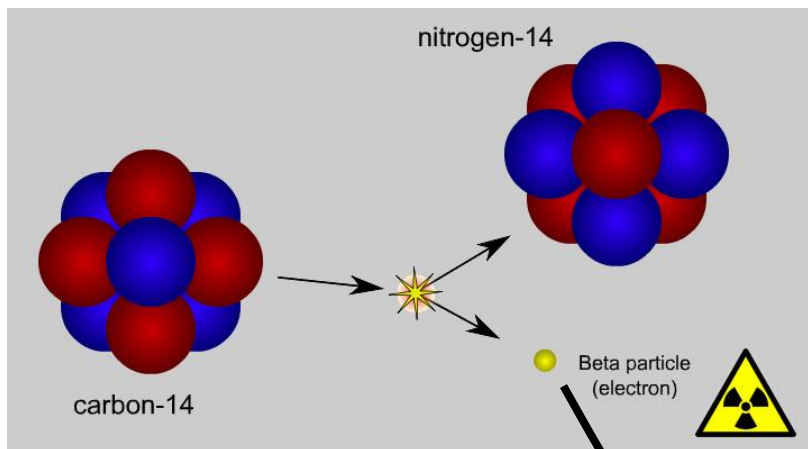


Schrödinger cat – the ultimate macroscopic quantum phenomenon

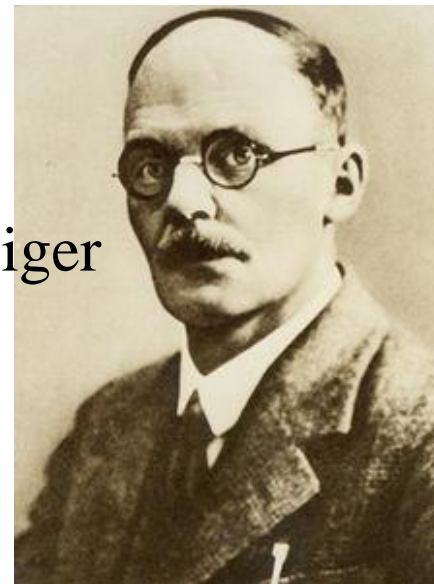
E. Schrödinger, Naturwiss. **23** (1935), 807.



Schrödinger cat – thought experiment



Hans Geiger



Geiger counter



Linearity of the Schrödinger's equation



Suppose Ψ_1 is a valid solution of the Schrödinger equation:

$$i\hbar \frac{\partial \psi_1}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi_1}{\partial x^2} + U(x)\psi_1$$

And suppose that Ψ_2 is another valid solution of the Schrödinger equation:

$$i\hbar \frac{\partial \psi_2}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi_2}{\partial x^2} + U(x)\psi_2$$

Then $(\Psi_1 + \Psi_2)/\sqrt{2}$ is also a valid solution, because:

$$i\hbar \frac{\partial (\psi_1 + \psi_2)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 (\psi_1 + \psi_2)}{\partial x^2} + U(x)(\psi_1 + \psi_2)$$

The state $(\Psi_1 + \Psi_2)/\sqrt{2}$ is a new combined state which is called “quantum superposition” of state (1) and (2)

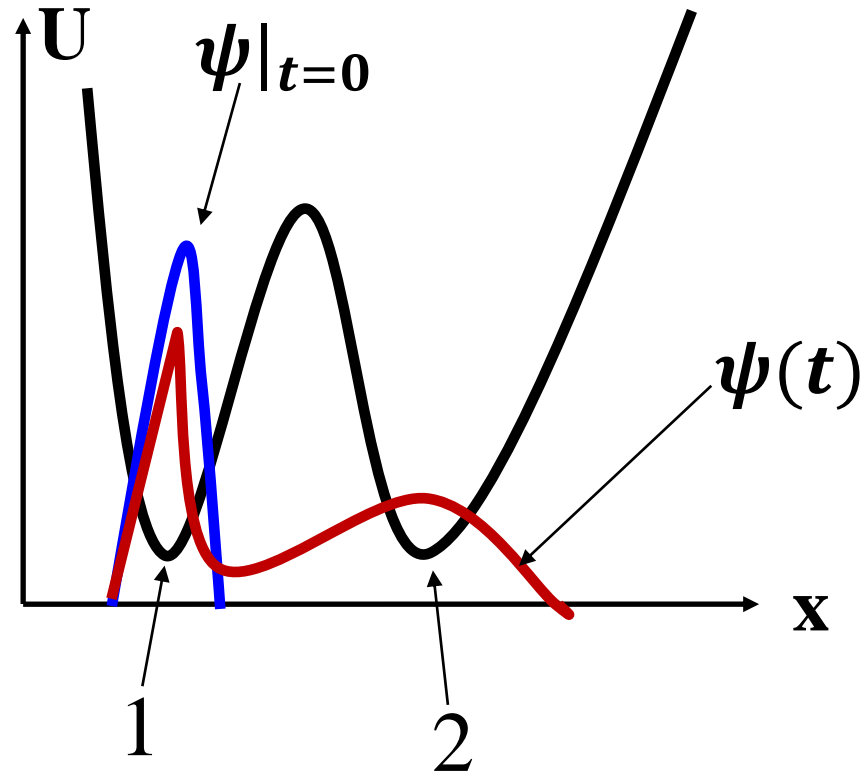


Quantum tunneling



George Gamow

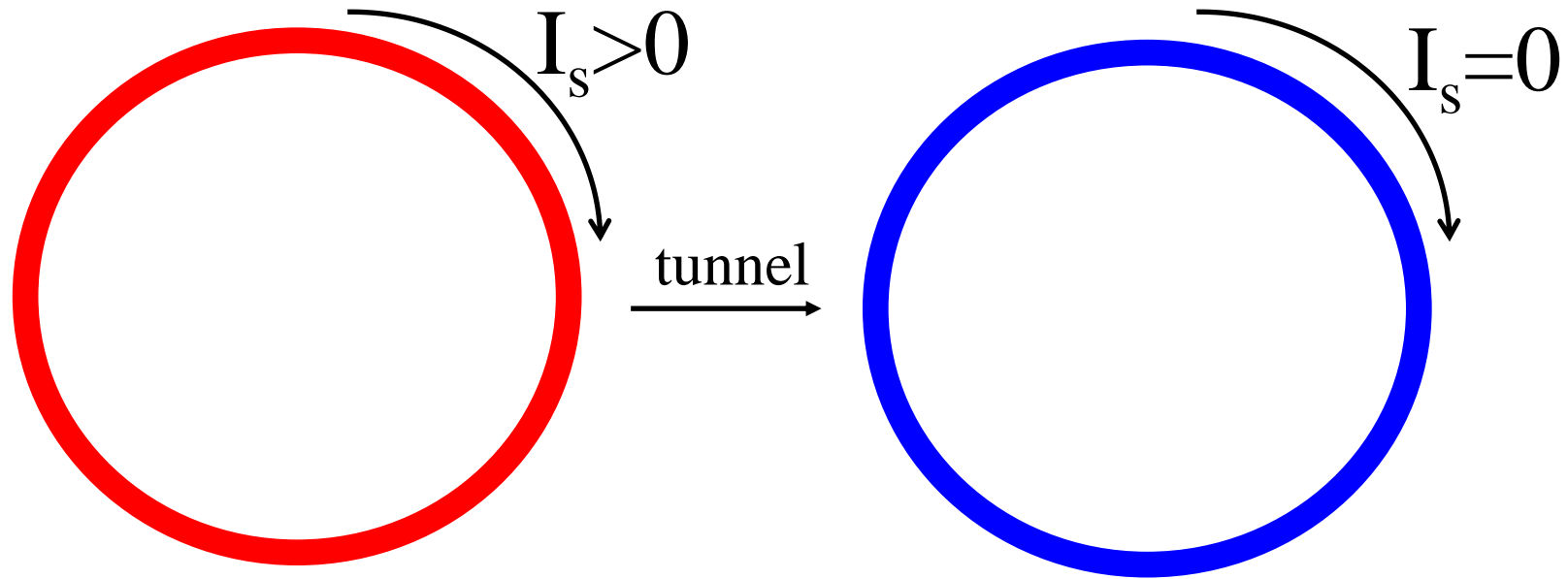
(He also helped to
developed
Big Bang theory)



**Quantum tunneling is possible
since quantum superpositions of
states are possible.**

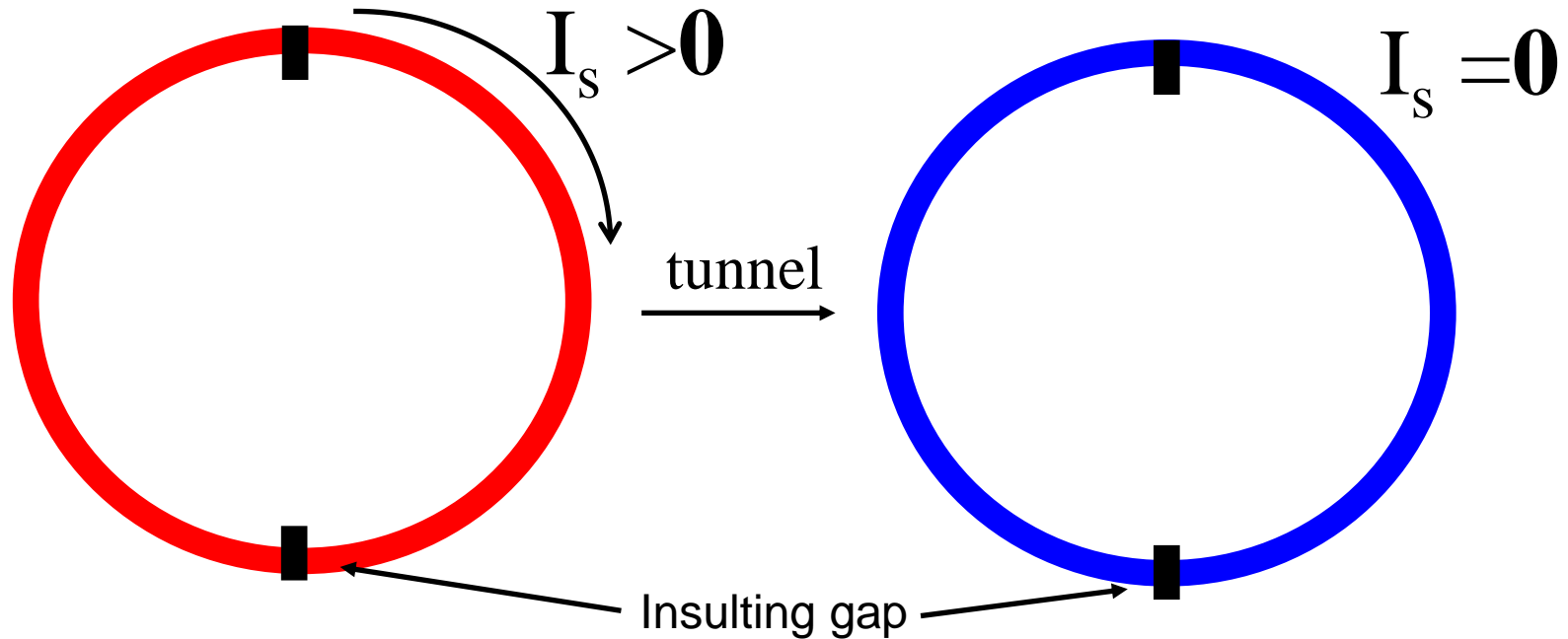


What sort of tunneling we will consider?



- Red color represents some strong current in the superconducting wire loop
- Blue color represents zero current in the loop

Previous results relate loops with insulating interruptions (SQUIDS)



- Red color represents some strong current in the superconducting loop
- Blue color represents zero current in the superconducting loop

Leggett's prediction for macroscopic quantum tunneling (MQT) in SQUIDs

80

Supplement of the Progress of Theoretical Physics, No. 69, 1980

Macroscopic Quantum Systems and the Quantum Theory of Measurement

A. J. LEGGETT

*School of Mathematical and Physical Sciences
University of Sussex, Brighton BN1 9QH*

(Received August 27, 1980)

It is this property which makes a SQUID the most promising candidate to date for observing macroscopic quantum tunnelling; if it should ever become possible to observe macroscopic quantum *coherence*, the low entropy and consequent lack of dissipation will be absolutely essential.²¹⁾



Types of Qubit



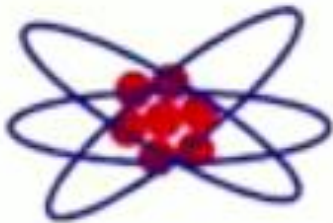
single spin-1/2

Quantum state:

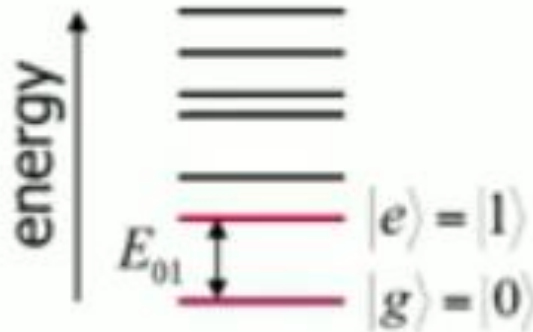
$$|\psi\rangle = A^*|0\rangle + B^*|1\rangle$$

$$A^2 + B^2 = 1$$

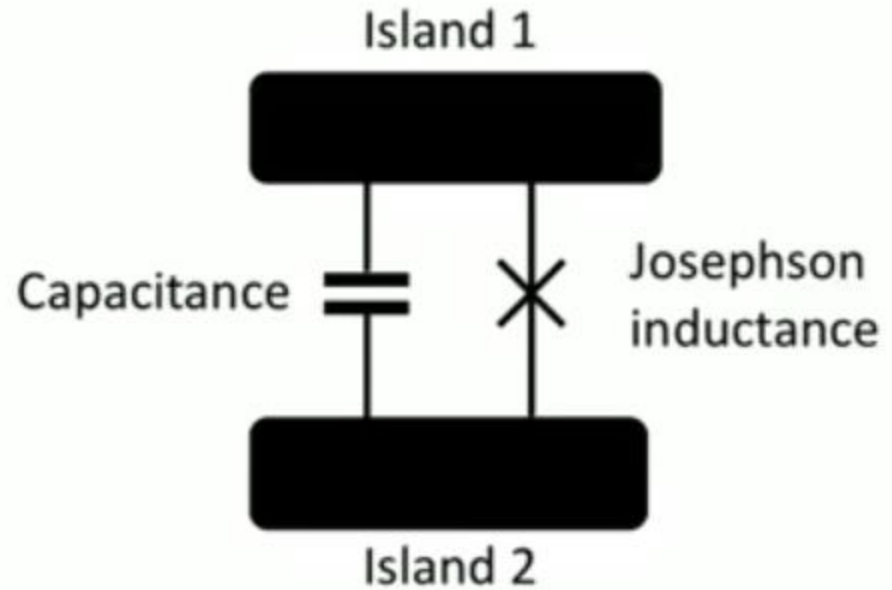
A and B are
complex numbers



single atom



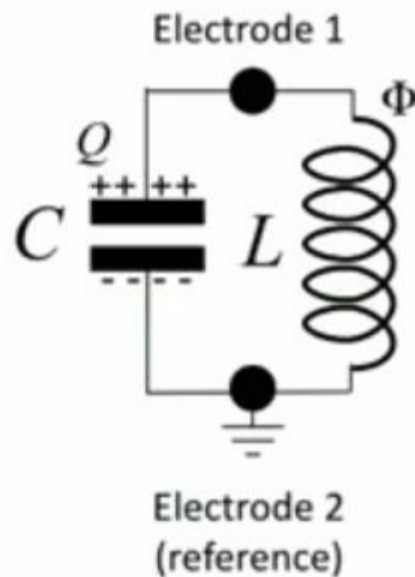
Transmon Qubit



Theory of transmons: J. Koch et al., Phys. Rev. A **76**, 042319 (2007).

Quantization of electrical circuits

The quantized LC oscillator



Hamiltonian:

$$\hat{H}_{LC} = \frac{\hat{Q}^2}{2C} + \frac{\hat{\Phi}^2}{2L}$$

Capacitive term Inductive term

Canonically conjugate variables:

$\hat{\Phi}$ = Flux through the inductor.

\hat{Q} = Charge on capacitor plate.

$$[\hat{\Phi}, \hat{Q}] = i\hbar$$

Discrete energy spectrum of the LC-circuit

Correspondence with simple harmonic oscillator

$$\hat{H}_{\text{LC}} = \frac{\hat{\Phi}^2}{2L} + \frac{\hat{Q}^2}{2C}$$

$$[\hat{\Phi}, \hat{Q}] = i\hbar$$

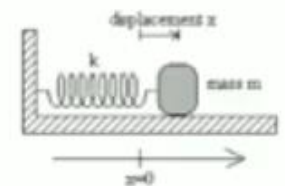
$$\hat{H}_{\text{SHO}} = \frac{k\hat{X}^2}{2} + \frac{\hat{P}^2}{2m}$$

$$[\hat{X}, \hat{P}] = i\hbar$$

Correspondence:

$$\begin{aligned} \hat{\Phi} &\leftrightarrow \hat{X} & L &\leftrightarrow \frac{1}{k} \\ \hat{Q} &\leftrightarrow \hat{P} & C &\leftrightarrow m \end{aligned}$$

$$\omega = \frac{1}{\sqrt{LC}} \leftrightarrow \sqrt{\frac{k}{m}}$$



Solve using ladder operators:

$$\hat{a} = \left(\frac{\hat{Q}}{Q_{\text{zpf}}} - i \frac{\hat{\Phi}}{\Phi_{\text{zpf}}} \right)$$

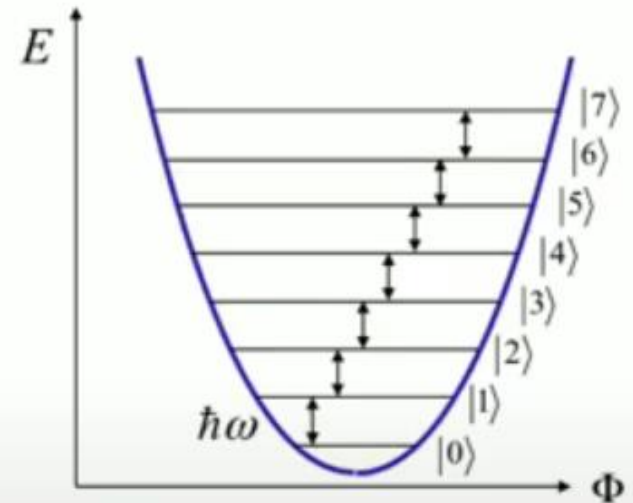
$$\Phi_{\text{zpf}} = \sqrt{2\hbar Z}$$

$$Q_{\text{zpf}} = \sqrt{2\hbar / Z}$$

$$\hat{a}^\dagger = \left(\frac{\hat{Q}}{Q_{\text{zpf}}} + i \frac{\hat{\Phi}}{\Phi_{\text{zpf}}} \right)$$

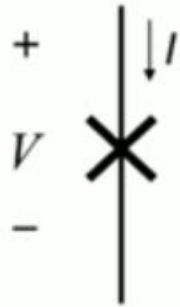
$$Z = \omega L = \frac{1}{\omega C} = \sqrt{\frac{L}{C}}$$

$$\hat{H}_{\text{LC}} = \hbar\omega \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right) \quad [\hat{a}_r, \hat{a}_r^\dagger] = 1$$



Non-harmonicity is the key factor

The Josephson junction



$$I = I_c \sin\left(2\pi \frac{\Phi}{\Phi_0}\right)$$

$$\Phi_0 = \frac{h}{2e}$$

flux quantum

$$V = \dot{\Phi}$$

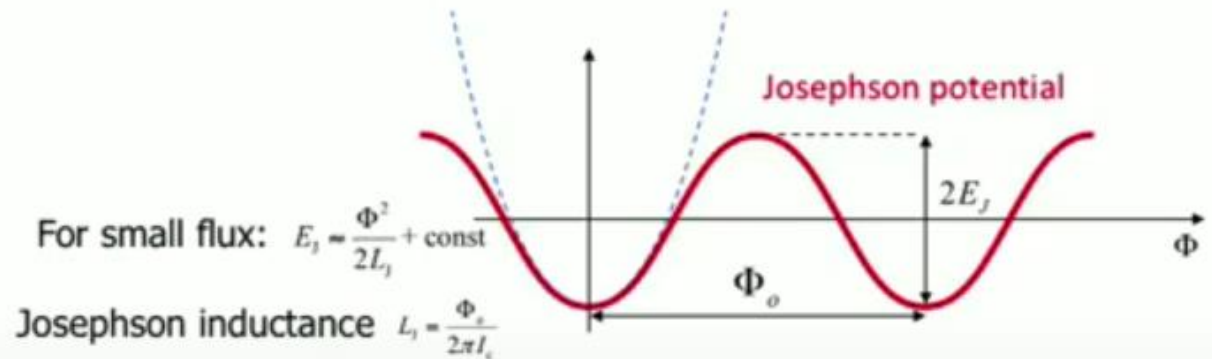


S superconductor-
I insulator-
S superconductor
tunnel junction

$$I_c = \frac{\pi \Delta}{2e R}$$

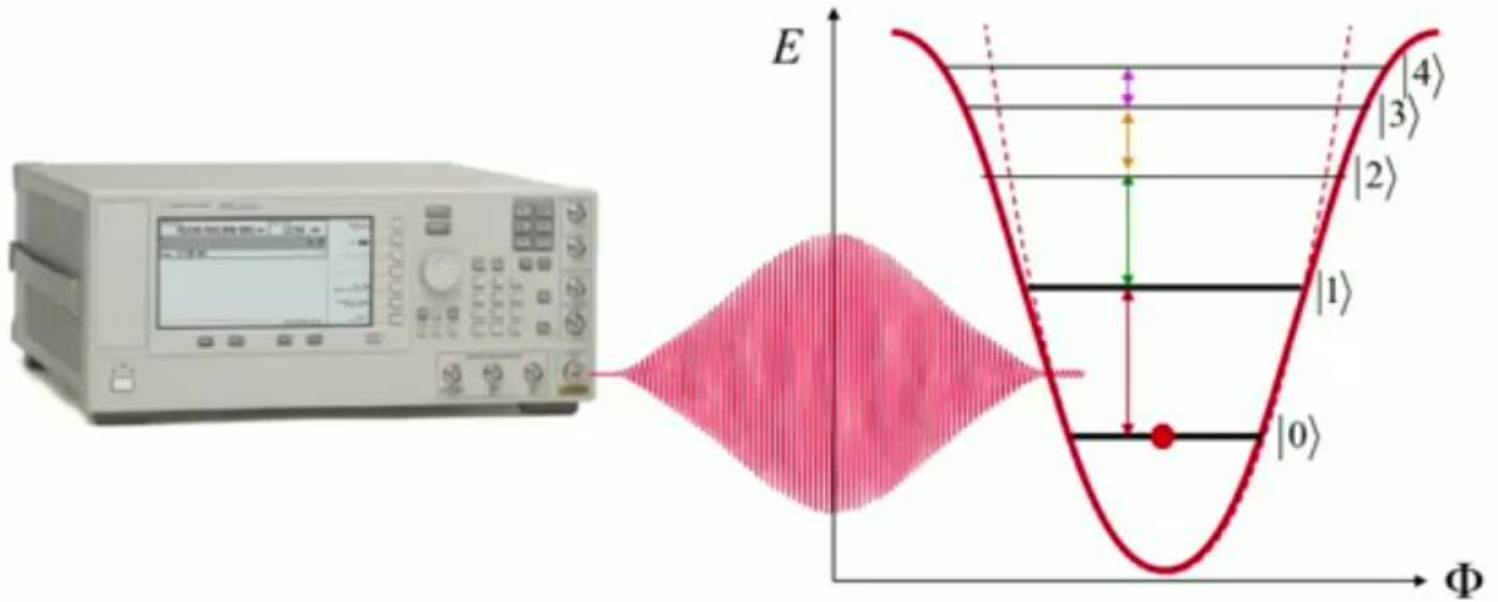
$$E_{\text{stored}} = E_J \left(1 - \cos\left(2\pi \frac{\Phi}{\Phi_0}\right)\right)$$

$$E_J = \frac{I_c \Phi_0}{2\pi} \quad \text{Josephson Energy}$$

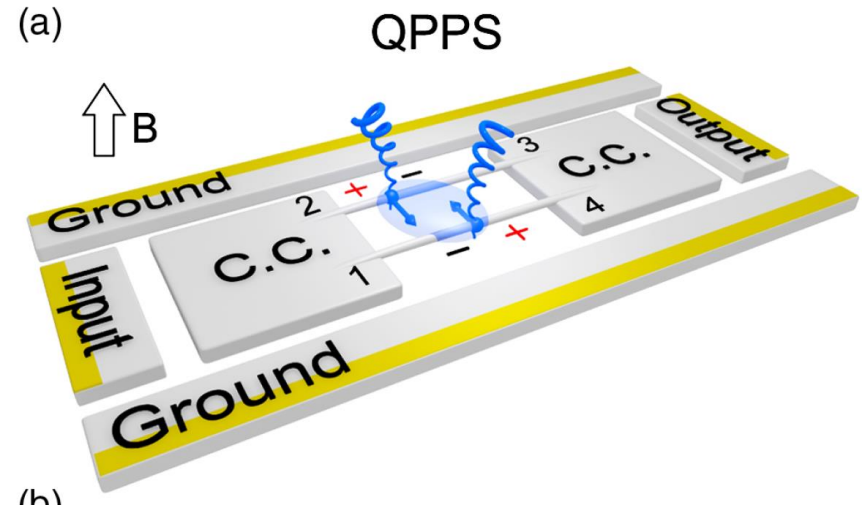
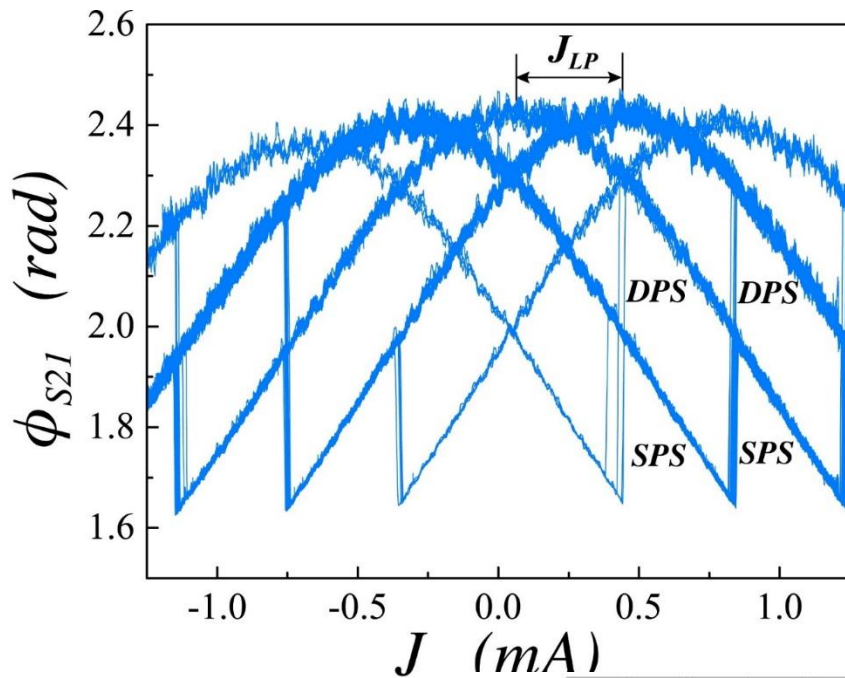


Non-harmonicity is the key factor

Transmon energy spectrum

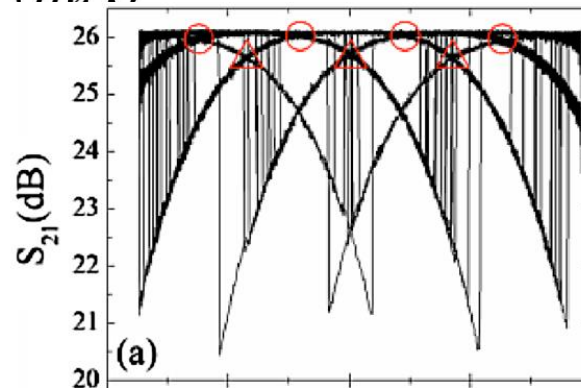


Superconducting nanowire memory: microwave readout



A. Belkin et al, PRX **5**, 021023 (2015)

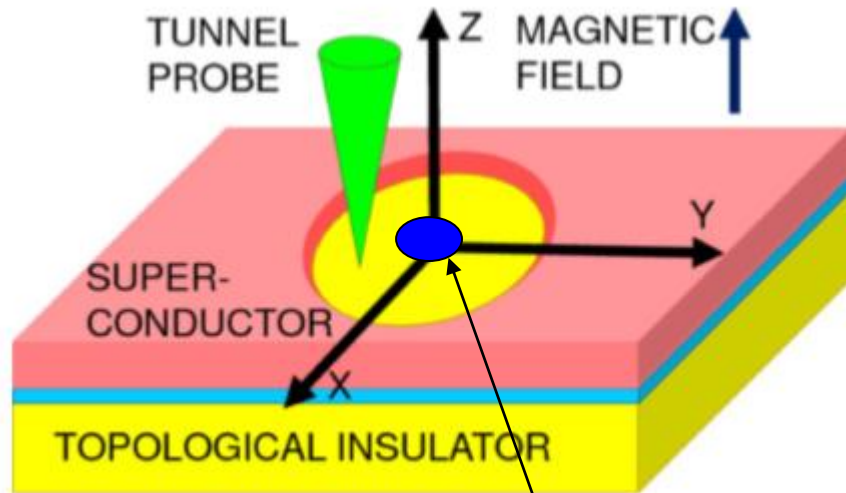
$T = 360$ mK
 $f = f_0(H=0)$



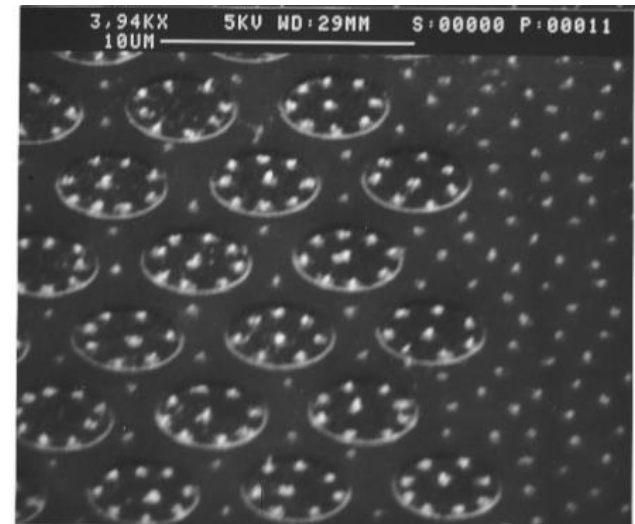
Andrey Belkin,^a Matthew Brenner,
 Thomas Aref, Jaseung Ku, and Alexey
 Bezryadin,
 PPLIED PHYSICS LETTERS **98**, 242504
 (2011)



Majorana modes in a vortex



Theory: Vortex in the nano-hole contains Majorana states (gap is about T_c)



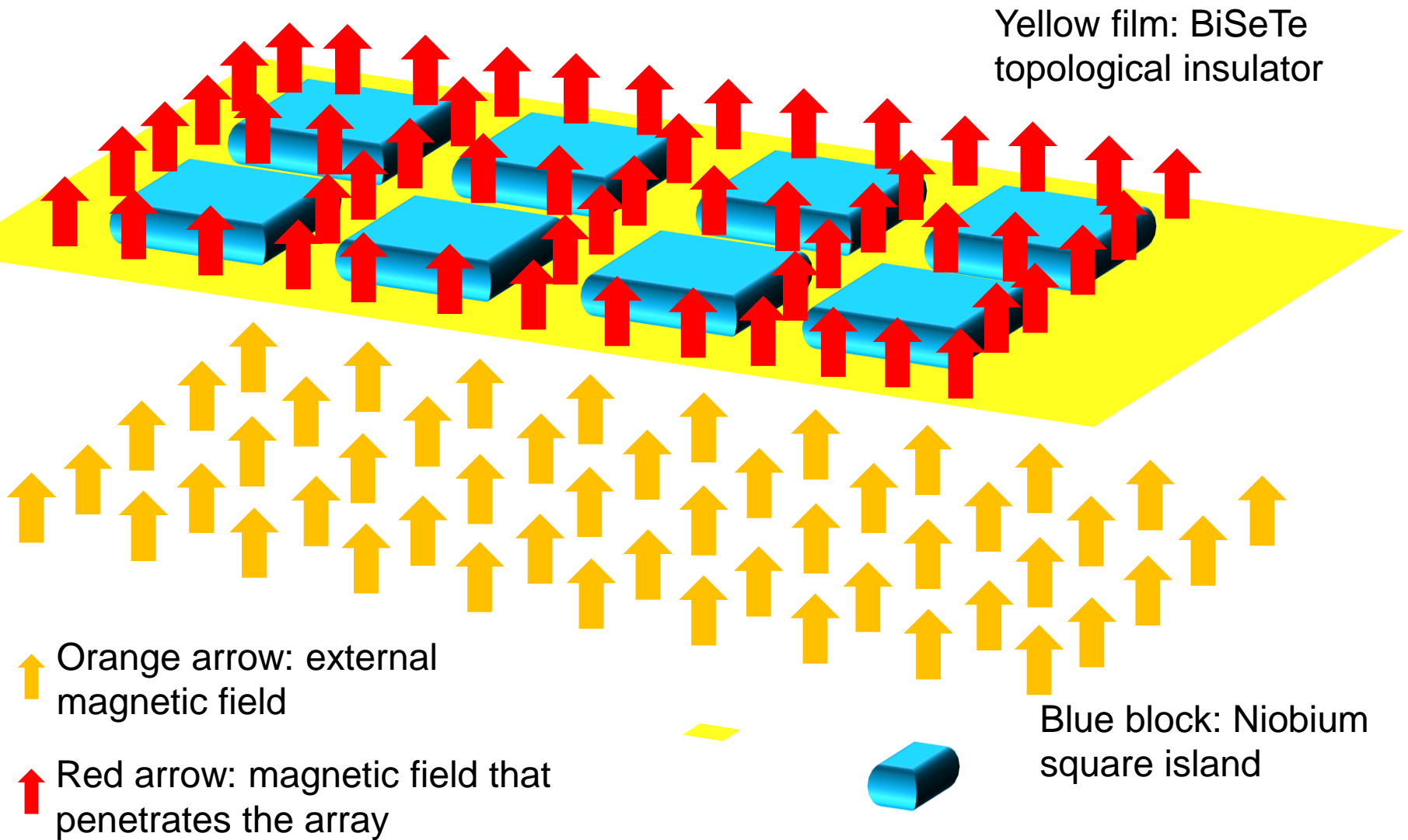
A. Bezryadin, Yu. Ovchinnikov, B. Pannetier, PRB 53, 8553 (1996)

R.S. Akzyanov, A.V. Rozhkov, A.L.Rakhmanov, and F. Nori, PRB 89, 085409 (2014)

PHYSICAL REVIEW B 84, 075141 (2011)



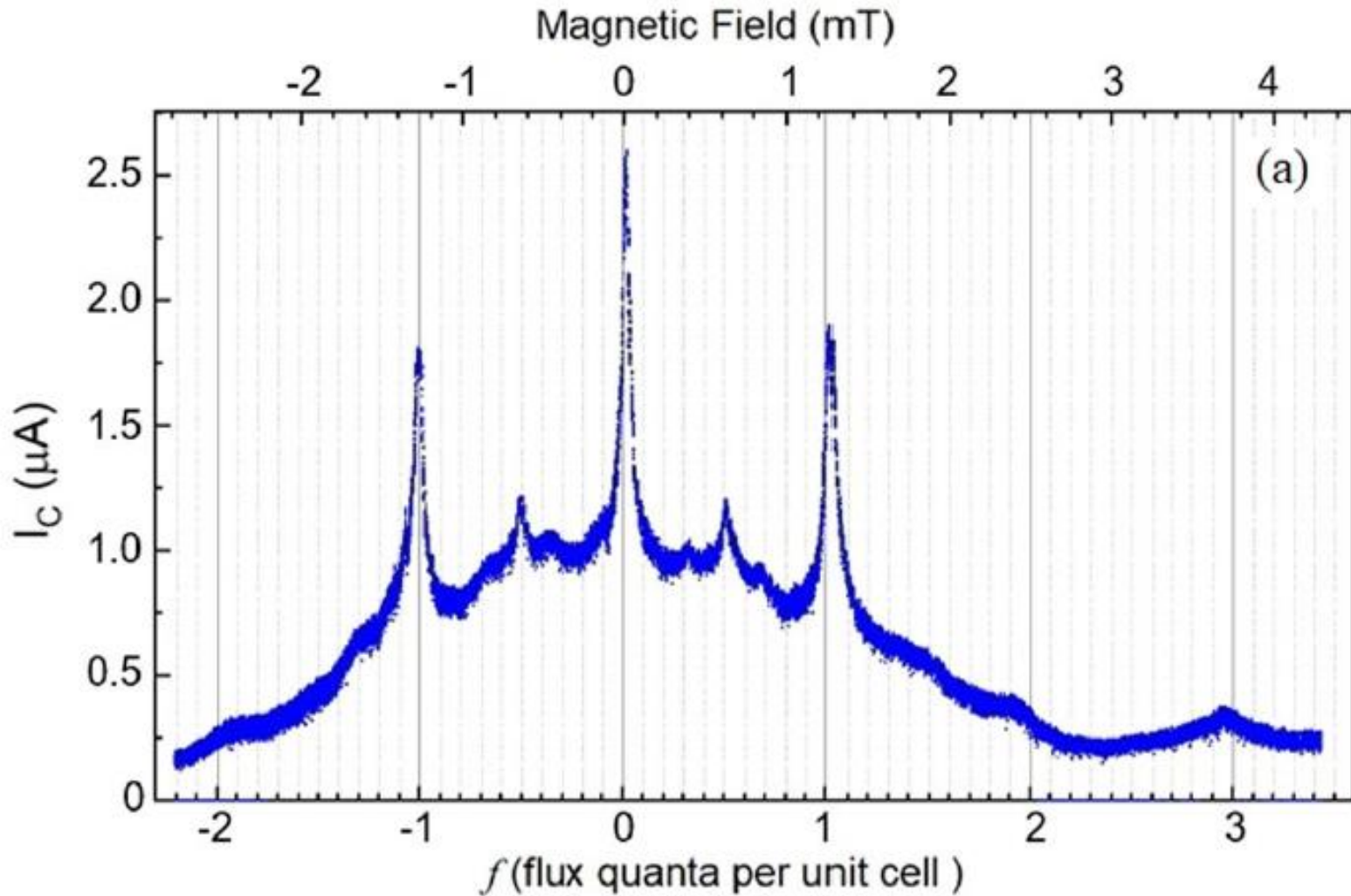
Superconducting array based on topological insulator- BST



Xiangyu Song, Soorya Suresh Babu, Yang Bai, Dmitry S. Golubev, Irina Burkova, Alexander Romanov, Eduard Ilin, James N. Eckstein & Alexey Bezryadin

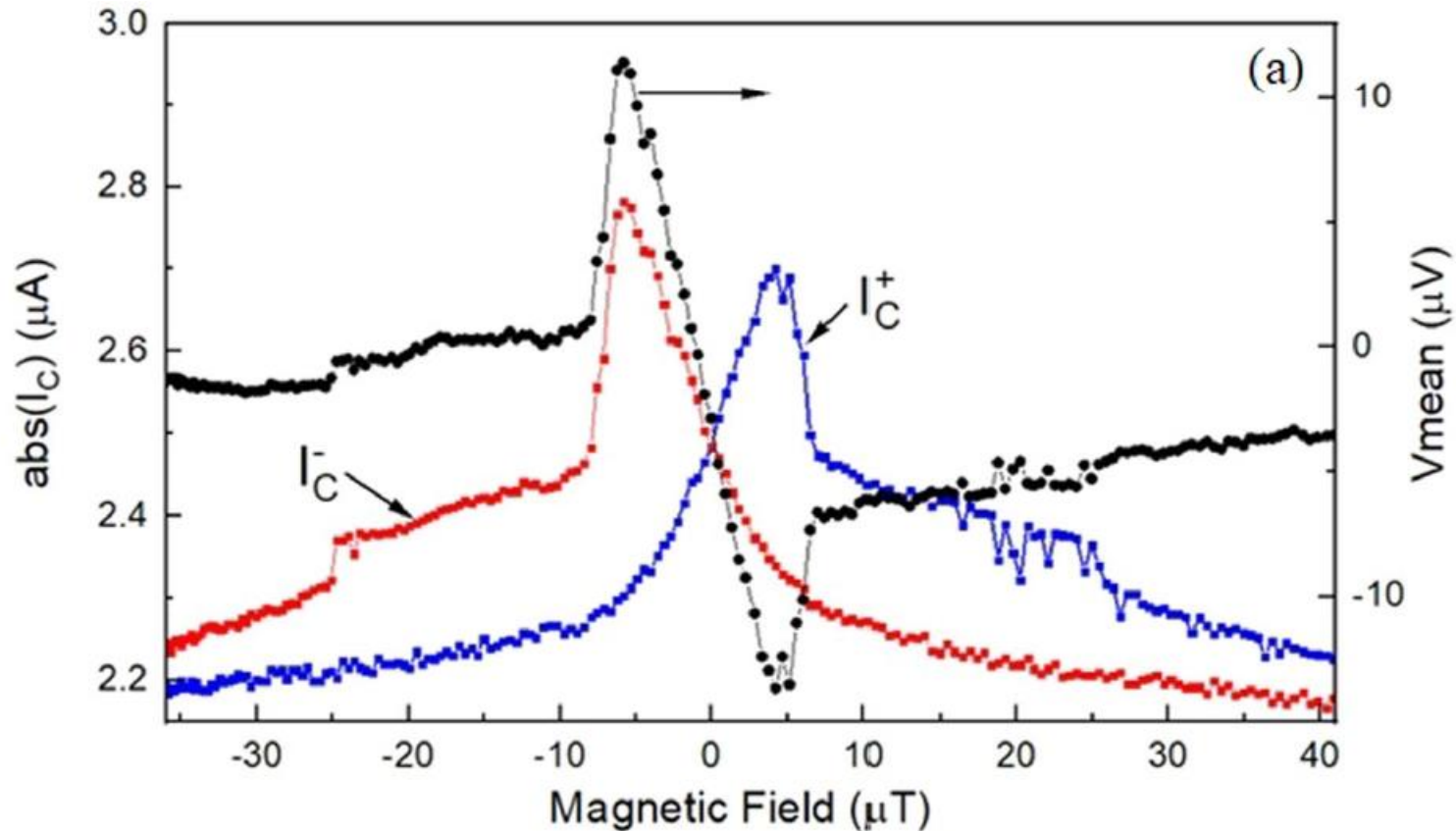
“Interference, diffraction, and diode effects in superconducting array based on bismuth antimony telluride topological insulator”
COMMUNICATIONS PHYSICS | (2023) 6:177 | <https://doi.org/10.1038/s42005-023-01288-9> | www.nature.com/commsphys

Diffraction grating analogy with superconducting arrays



Xiangyu Song, Soorya Suresh Babu, Yang Bai, Dmitry S. Golubev, Irina Burkova, Alexander Romanov, Eduard Ilin, James N. Eckstein & Alexey Bezryadin
“Interference, diffraction, and diode effects in superconducting array based on bismuth antimony telluride topological insulator”
COMMUNICATIONS PHYSICS | (2023) 6:177 | <https://doi.org/10.1038/s42005-023-01288-9> | www.nature.com/commsphys

Diode effect with superconducting arrays

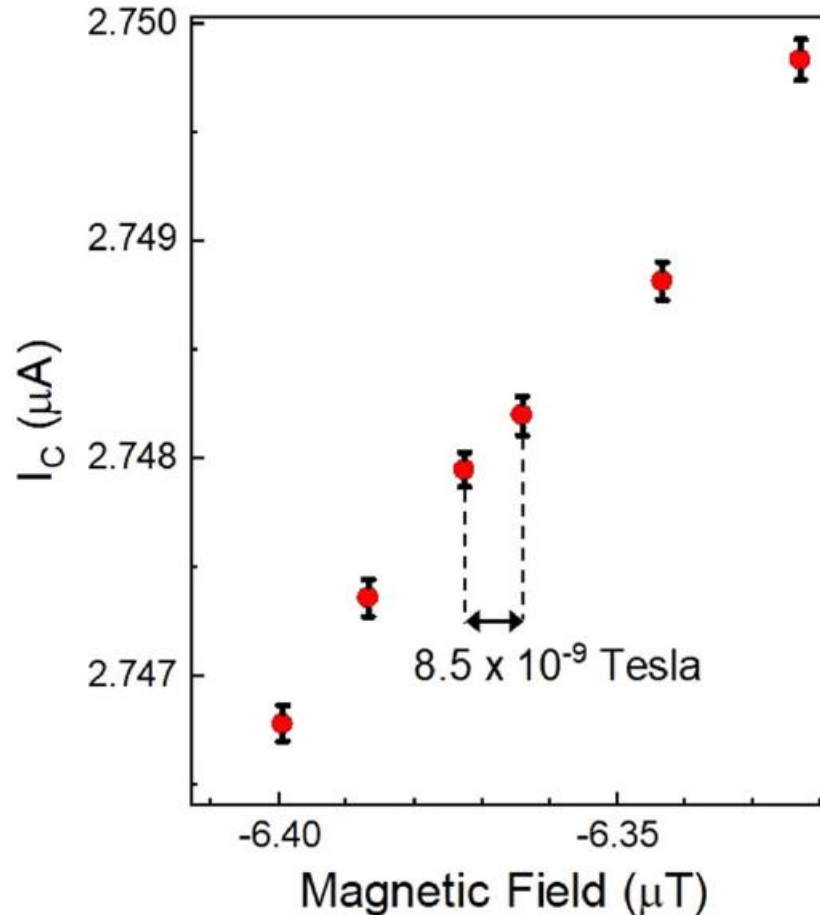


Xiangyu Song, Soorya Suresh Babu, Yang Bai, Dmitry S. Golubev, Irina Burkova, Alexander Romanov, Eduard Ilin, James N. Eckstein & Alexey Bezryadin

“Interference, diffraction, and diode effects in superconducting array based on bismuth antimony telluride topological insulator”

COMMUNICATIONS PHYSICS | (2023) 6:177 | <https://doi.org/10.1038/s42005-023-01288-9> | www.nature.com/commsphys

Absolute magnetic field sensor based on superconducting array



Xiangyu Song, Soorya Suresh Babu, Yang Bai, Dmitry S. Golubev, Irina Burkova, Alexander Romanov, Eduard Ilin, James N. Eckstein & Alexey Bezryadin
“Interference, diffraction, and diode effects in superconducting array based on bismuth antimony telluride topological insulator”
COMMUNICATIONS PHYSICS | (2023) 6:177 | <https://doi.org/10.1038/s42005-023-01288-9> | www.nature.com/commsphys

Conclusions

- Superconductivity is related to fundamental quantum phenomena. We have reviewed some of them. They will be discussed in more details in the future lectures.
- Superconductors have been used to create strong and stable magnetic fields, in levitating trains for example.
- Superconducting quantum interference devices enabled researches to measure very small magnetic fields, such as those produced by human brain.
- Superconductors are used to build qubits, which are the building blocks of quantum computers.

